Eur. Phys. J. C 8, 689–695 (1999)
DOI 10.1007/s100529901110

THE EUROPEAN PHYSICAL JOURNAL C

The quark mass correction to the string potential

G. Germán[1], Yu. Jiang[2]

[1] Instituto de Física, Laboratorio de Cuernavaca, Universidad Nacional Autónoma de México, Apartado Postal 48-3, 62251 Cuernavaca, Morelos, México

[2] Facultad de Ciencias, Universidad Autónoma del Estado de Morelos, Av. Universidad 1001, Col. Chamilpa, 62210 Cuernavaca, Morelos, México

Received: 21 April 1998 / Revised version: 19 November 1998 / Published online: 27 April 1999

Abstract. Following recent work by Lambiase and Nesterenko we study in detail the interquark potential for a Nambu–Goto string with point masses attached to its ends. We obtain accurate solutions to the gap equations for the Lagrange multipliers and metric components and determine the potential without simplifying assumptions. We also discuss the Lüscher term and argue that it remains universal.

1 Introduction

There has been considerable effort in trying to understand the forces between quarks in terms of strings, and several models have been proposed with different degrees of success. The Nambu–Goto model [1], which is a direct generalization of the covariant action for a relativistic point particle moving in space-time, describes the evolution of a string. When this string evolves it sweeps out a two-dimensional world sheet surface embedded in a higher-dimensional space-time. The area of this surface is precisely the Nambu–Goto action. Close to this are the generalized Eguchi models [2] of which Schild's [3] is a particular case. The functional-integral quantization of these models has been studied by Lüscher, Symanzik and Weisz [4] and by Alvarez [5], who calculated the static potential in the large-d limit, where d is the number of dimensions of the embedding space. The result obtained by Alvarez turned out to be correct for any d as shown latter by Arviz [6]. The Nambu–Goto string model gives qualitatively encouraging results as a large-N QCD string, i.e., the interquark potential is linear for large distances, which is understood as a signal of confinement; it also has linear Regge trajectories and presents a transition to a deconfined phase with vanishing string tension at certain temperature [7]. Quantitatively, however, this model is not in very good shape, with numerical values closer to those obtained in Monte Carlo simulations of an SU(2) lattice gauge theory rather than SU(3) [8]. Also, it has been shown [9] that agreement between the Nambu–Goto string and a calculation of the high-temperature partition function of a QCD flux tube would require an infinite number of massive world-sheet degrees of freedom. Thus, the model has been modified by populating the string with scalar and Fermi fields, improving some of the quantitative results [10]. However, the price to pay is too high since the conformal invariance of the theory is explicitly broken [11]. It seems that the Nambu–Goto string or naive modifications of it will not give us the QCD string. Still the Nambu–Goto model remains very useful as the simplest string model where some calculations can be done without undue effort and mathematical methods as well as new physical ideas can be tested. More elaborated extensions of Nambu–Goto have incorporated an extrinsic curvature term in the action (the so-called rigid or Polyakov–Kleinert string [12]) and several properties of interest have been investigated [13]. Rigid strings coupled to long-range Kalb–Ramond fields have also been studied [14], and more recently "confining strings" [15] seem to be very promising models for the QCD string. In all of these models one important problem is to determine the potential between two sources, i.e., the so-called interquark static potential. This potential has been calculated by various perturbative and non-perturbative methods. The common feature has been, however, the assumption of infinitely massive quarks at the ends of the string, which is equivalent to imposing fixed-ends boundary conditions. In a recent series of papers a consistent method has been proposed to study the effects of finite point masses attached to the ends of the string [16]. In particular a variational estimation of the Nambu–Goto string potential has been worked out although with some simplifying assumptions[17] and a more general discussion followed [18].

Here we reconsider this problem and a detailed treatment is presented. Accurate solutions to the gap equations and a determination of the interquark potential as well as other quantities of interest are given. Discrepancies with respect to the original work of [17] are pointed out. We also provide a discussion of the Lüscher term and argue that it remains universal with no mass contributions coming from the point particles attached to the ends of the string, in disagreement with what is claimed in [17]. In Sect. 2 we present the model and equations for the Lagrange multipliers and metric components. We also obtain a very simple-looking formula for the interquark potential. The numerical analysis of the equations and various quantities of interest is carried out in Sect. 3. We also compare them with the approximated results in [17]. Finally, Sect. 4 comprises a discussion of the Lüscher term and argues that it remains universal. We conclude with a brief account of our results.

2 The model and gap equations

At the quantum level the Nambu–Goto model is given by the functional integral

$$Z = \int [Dx^\mu] \mathrm{e}^{-S}. \tag{2.1}$$

In Euclidean space the action S is

$$S = M_0^2 \int \mathrm{d}^2\xi \sqrt{g} + \sum_{a=1}^{2} m_a \int_{C_i} \mathrm{d}s_a, \tag{2.2}$$

where M_0^2 is the string tension, C_i $(i = 1, 2)$ are the world trajectories of the massive ends of the string, and g is the determinant of the metric

$$g_{ij} = \partial_i x^\mu(\xi_i)\partial_j x^\nu(\xi_i)\eta_{\mu\nu}, \qquad i = 0, 1. \tag{2.3}$$

The x^μ, $\mu = 0, 1, ..., d-1$ are the string coordinates and $\eta_{\mu\nu}$ is the embedding Euclidean metric of the space where the string evolves. g^{ij} is thus the induced metric on the world sheet swept out by the string. To study the model further it is convenient to specify a gauge; we choose the "physical gauge" or Monge parametrization

$$x^\mu(\xi_i) = (t, r, u^a(t, r)), \tag{2.4}$$

where the $\vec{u}^a(t, r)$, $a = 2, ..., d-1$ are the $(d-2)$ transverse oscillations of the string. We further introduce composite fields σ_{ij} given by

$$\sigma_{ij} = \partial_i \vec{u} \cdot \partial_j \vec{u}. \tag{2.5}$$

The metric g_{ij} and string coordinates $\vec{u}$ become independent fields when (2.3) is introduced as a constraint. This requires the use of Lagrange multipliers α^{ij} which also become independent variables. The functional integral (2.1) then becomes

$$Z = \int [D\vec{u}][D\alpha][D\sigma] \mathrm{e}^{-S(\vec{u},\alpha,\sigma)}, \tag{2.6}$$

where the action (2.2) is now given by

$$\begin{aligned} S = M_0^2 \int_0^\beta \mathrm{d}t \int_0^R \mathrm{d}r & \left[\sqrt{\det(\delta_{ij} + \sigma_{ij})} \right. \\ & \left. + \frac{1}{2}\alpha^{ij}(\partial_i \vec{u} \cdot \partial_j \vec{u} - \sigma_{ij}) \right] \\ & + \sum_{a=1}^{2} m_a \int \mathrm{d}t \sqrt{1 + \dot{\vec{u}}^2(t, r_a)}, \quad r_1 = 0,\ r_2 = R. \end{aligned} \tag{2.7}$$

It has been shown by Alvarez that, at the saddle point, the Lagrange parameters α^{ij} as well as the metric components σ_{ij} become symmetric constant matrices with no dependence on t and r. Thus while $\vec{u} = \vec{u}(t, r)$ is in general a function of t and r, $\dot{\vec{u}}^2 = \sigma_0$ becomes, at the saddle point, a constant. This fact simplifies the problem considerably. Since the action is quadratic in the string oscillations $\vec{u}^a$ we can do the Gaussian integral inmediately. The resulting action, in the particular case where $m_1 = m_2 = m$, can be written as

$$\begin{aligned} S(\alpha, \sigma) = M_0^2 \beta R & \left[\sqrt{(1 + \sigma_0)(1 + \sigma_1)} - \frac{1}{2}(\alpha_0\sigma_0 + \alpha_1\sigma_1) \right. \\ & \left. - \sqrt{\frac{\alpha_1}{\alpha_0}}\lambda \right] + 2m\beta. \end{aligned} \tag{2.8}$$

Here λ is related to the Casimir energy $E_\mathrm{C} = \frac{1}{2}\sum_{k=1}^\infty \omega_k$ as follows

$$\lambda = -\frac{(D-2)}{M_0^2 R} E_\mathrm{C}, \tag{2.9}$$

and the last term in (2.8) is the contribution to the action due to the point masses at the ends of the string. This term can be set to zero with an appropriate redefinition of S. Thus we ignore this term in what follows. The Casimir energy E_C depends on the eigenmomenta ω_k which in turn depend on the boundary conditions imposed on the system. For a string with infinitely heavy quarks attached to its ends we impose fixed-ends boundary conditions, in this case

$$\omega_k = \frac{n\pi}{R}, \qquad n = 1, 2, ..., \tag{2.10}$$

and the Casimir energy is

$$E_\mathrm{C} = \frac{1}{2}\sum_{k=1}^\infty \omega_k = \frac{\pi}{2R}\sum_{n=1}^\infty n = -\frac{\pi}{24R}, \tag{2.11}$$

where the last term was obtained by the use of Riemann's ζ-function i.e.,

$$\sum_{n=1}^\infty n = \left[\sum_{n=1}^\infty \frac{1}{n^\nu}\right]_{\nu=-1} = \zeta(-1) = -\frac{1}{12}.$$

In the case of finite quark masses the problem becomes increasingly difficult to deal with even when $m_1 = m_2$. It can be shown that in this case $(m_1 = m_2 = m)$ the Casimir energy is given by [17]

$$E_\mathrm{C} = \frac{1}{2\pi R}\int_0^\infty \mathrm{d}x \ln\left[1 - \left(\frac{x - s}{x + s}\right)^2 \mathrm{e}^{-2x}\right], \tag{2.12}$$

where

$$s = \frac{\rho}{\mu}\alpha_0\sqrt{1 + \sigma_0}, \tag{2.13}$$

and

$$\rho = M_0 R, \qquad \mu = \frac{m}{M_0} \tag{2.14}$$

are dimensionless quantities corresponding to the (extrinsic) length and point masses attached to the ends of the string, respectively. The equation for λ, (2.9), becomes

$$\lambda = -\frac{(D-2)}{2\pi\rho^2}\eta(s), \tag{2.15}$$

where

$$\eta(s) = \int_0^\infty \mathrm{d}x \ln\left[1 - \left(\frac{x-s}{x+s}\right)^2 \mathrm{e}^{-2x}\right]. \tag{2.16}$$

This expression for η can be written as

$$\eta(s) = L_{i2}(-1/s) - 1/2\sum_{i=1}^{3} L_{i2}(1/r_i), \tag{2.17}$$

where $L_{i2}(1/r_i)$ are dilogarithm functions and r_i are the roots of

$$x^3 - (1+2s)x^2 + s(s-2)x - s^2 = 0. \tag{2.18}$$

Note that

$$\mathrm{L}_{i2}(x) = -\int_0^1 \frac{\ln(1-xt)}{t}\mathrm{d}t = -\int_0^x \frac{\ln(1-t)}{t}\mathrm{d}t. \tag{2.19}$$

It has a branch cut running from 1 to ∞. It is also convenient to write λ in the form

$$\begin{aligned}\lambda = {} & \frac{(D-2)\pi}{24\rho^2} \\ & -\frac{(D-2)}{2\pi\rho^2}\int_0^\infty \mathrm{d}x \ln\left[1 + \frac{4sx}{(x+s)^2}\frac{1}{\mathrm{e}^{2x}-1}\right].\end{aligned} \tag{2.20}$$

Note that λ is a function of α_0 and σ_0 through s, (2.13). Thus when writing the equations for the Lagrange multipliers and metric components derivatives of λ with respect to σ_0 and α_0 should appear. These are given by

$$\alpha_0 = \sqrt{\frac{1+\sigma_1}{1+\sigma_0}} - \frac{\sqrt{\alpha_0\alpha_1}}{1+\sigma_0}\frac{\partial\lambda}{\partial\alpha_0}, \tag{2.21a}$$

$$\alpha_1 = \sqrt{\frac{1+\sigma_0}{1+\sigma_1}}, \tag{2.21b}$$

$$\sigma_0 = \frac{1}{\alpha_0}\sqrt{\frac{\alpha_1}{\alpha_0}}\lambda - 2\sqrt{\frac{\alpha_1}{\alpha_0}}\frac{\partial\lambda}{\partial\alpha_0}, \tag{2.21c}$$

$$\sigma_1 = -\frac{1}{\sqrt{\alpha_0\alpha_1}}\lambda, \tag{2.21d}$$

where, in (2.21a), $\partial\lambda/\partial\sigma_0$ has been replaced by

$$\frac{\partial\lambda}{\partial\sigma_0} = \frac{\alpha_0}{2(1+\sigma_0)}\frac{\partial\lambda}{\partial\alpha_0}. \tag{2.22}$$

The potential $V(\rho)$ is obtained in the usual way, $\mathrm{e}^{-\beta V(\rho)} \sim Z$, $\beta \to \infty$, and is given by the simple-looking formula

$$\overline{V}(\rho) = \rho\alpha_0, \tag{2.23}$$

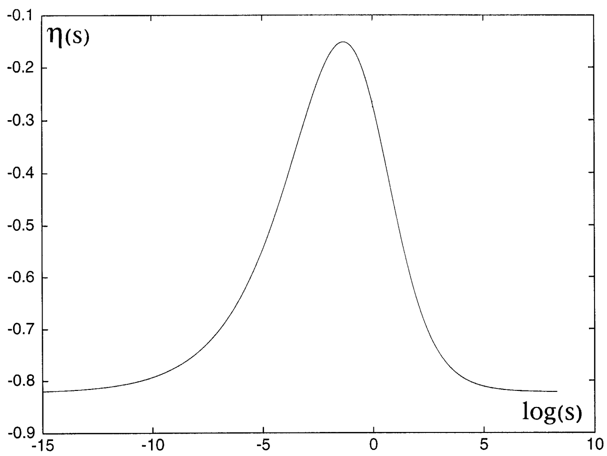

Fig. 1. $\eta(s)$ as a function of $\log(s)$, (2.16), which essentially defines the Casimir energy (2.12). For $\mu = 0, \infty$ the quantity s given by (2.13) takes the values ∞ and 0, respectively, and the Casimir energy becomes $E_\mathrm{C} = -\frac{\pi}{24R}$. This value coincides with the one obtained for a string with fixed-ends boundary conditions. We see that $\eta(s)$ has a maximum at $s = s_0 \approx 0.27$. At this point we can obtain an exact analytical solution given by (3.8)

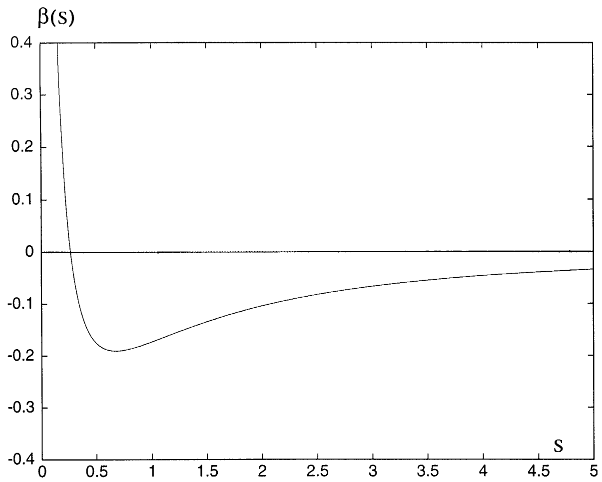

Fig. 2. The quantity $\beta = \frac{\partial\eta(s)}{\partial s}$ is shown as a function of s. The point $s = s_0 \approx 0.27$ where $\beta(s_0) = 0$ corresponds to the maxima of $\eta(s)$. From (3.2) we see that $c(s_0) = 0$; (3.5) implies $x = 1$ and a particular solution follows (see (3.8))

which follows from (2.8) and the gap equations (2.21). The potential $\overline{V}(\rho)$ is also a dimensionless quantity, $\overline{V}(\rho) = M_0^{-1}V(\rho)$. Of course there is no way to solve (2.21) analytically, thus (2.23) is only a formal expression for $\overline{V}(\rho)$. One can play with (2.21) and write an expression for α_0:

$$\alpha_0 = \sqrt{1 - \frac{1+\alpha_0\alpha_1}{\sqrt{\alpha_0\alpha_1}}\lambda - (1-\alpha_0\alpha_1)\sqrt{\frac{\alpha_0}{\alpha_1}}\frac{\partial\lambda}{\partial\alpha_0}}, \tag{2.24}$$

which will be useful for discussing some limiting situations in the last section.

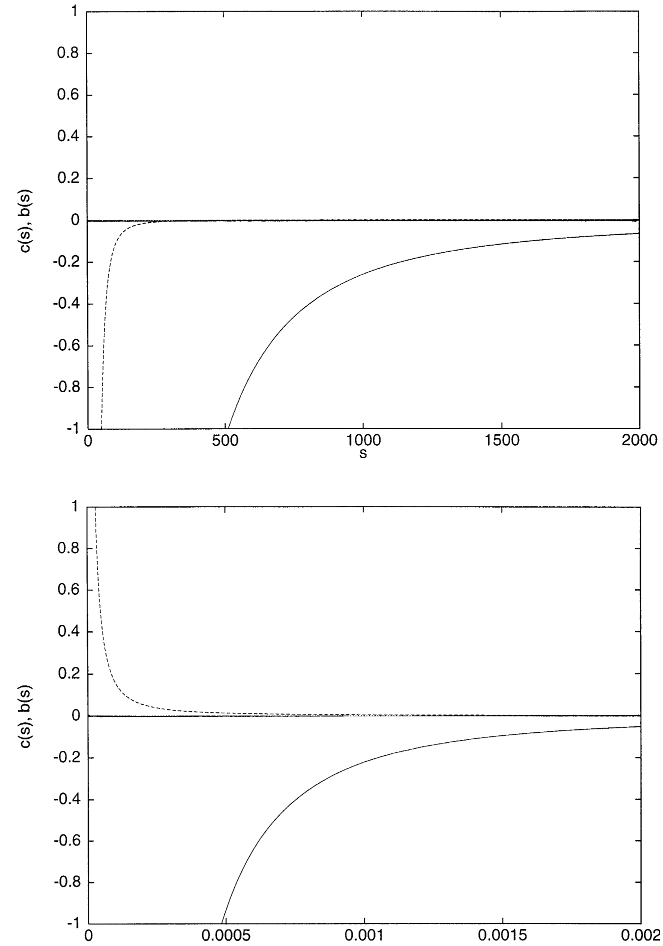

Fig. 3. $c(s)$ (*dashed line) and* $b(s)$ *(*solid line) as functions of s for $\mu = 10^{-3}$ (**a**) and 10^3 (**b**). These quantities are defined by (3.2) and (3.3), respectively. In **a** $c(s)$ eventually reaches a minimum value and then goes up passing through zero at $s = s_0$. The curve for $b(s)$ is always negative as follows from (3.3)

3 Numerical analysis

For the numerical analysis of the problem it is more convenient to write (2.21) in the form

$$\alpha_0 = \sqrt{\frac{1+\sigma_1}{1+\sigma_0}} + c\alpha_0\sqrt{\alpha_0\alpha_1}, \tag{3.1a}$$

$$\alpha_1 = \sqrt{\frac{1+\sigma_0}{1+\sigma_1}}, \tag{3.1b}$$

$$\sigma_0 = -\frac{\alpha_1\sigma_1}{\alpha_0} + 2c(1+\sigma_0)\sqrt{\alpha_0\alpha_1}, \tag{3.1c}$$

$$\sigma_1 = \frac{\alpha_0^2(1+\sigma_0)}{\sqrt{\alpha_0\alpha_1}}b, \tag{3.1d}$$

where

$$c = \frac{(D-2)}{2\pi}\frac{\beta(s)}{\mu^2 s}, \qquad \beta(s) = \frac{\partial\eta(s)}{\partial s}, \tag{3.2}$$

$$b = \frac{(D-2)}{2\pi}\frac{\eta(s)}{\mu^2 s^2}. \tag{3.3}$$

Combining (3.1a) and (3.1b) we get

$$\alpha_0\alpha_1 = 1 + c\alpha_0\alpha_1\sqrt{\alpha_0\alpha_1}, \tag{3.4}$$

or

$$cx^3 - x^2 + 1 = 0, \tag{3.5}$$

where

$$x = \sqrt{\alpha_0\alpha_1}. \tag{3.6}$$

We can now solve (3.1) in terms of x

$$\alpha_0 = \sqrt{\frac{1+(b-2c)x}{(1-(b+c)x)(1-cx)}}, \tag{3.7a}$$

$$\alpha_1 = \sqrt{\frac{1-(b+c)x}{(1+(b-2c)x)(1-cx)}}, \tag{3.7b}$$

$$\sigma_0 = -\frac{(b-2c)x}{1+(b-2c)x}, \tag{3.7c}$$

$$\sigma_1 = \frac{bx}{1-(b+c)x}. \tag{3.7d}$$

Thus, in the end, everything depends on s and μ. From (2.13) we can recover the ρ dependence. In Figs. 1–3 we have the behavior of η, β, c and b as functions of s for various values of the mass parameter μ. As a curiosity, where an exact analytical solution can be obtained, we see in Fig. 1 that $\eta(s)$ presents a maximum for $s = s_0 \approx 0.27$; at this point $\beta = \frac{\partial\eta(s)}{\partial s}$ vanishes and the problem can be solved exactly. This point corresponds to the maximum value of the Casimir energy (2.15) for a given length ρ. In this point $c(s_0) = 0$ and from (3.5) $x = 1$. Thus the solution to the gap equations is

$$\alpha_0 = \sqrt{1-2\lambda}, \tag{3.8a}$$

$$\alpha_1 = \frac{1}{\sqrt{1-2\lambda}}, \tag{3.8b}$$

$$\sigma_0 = \frac{\lambda}{1-2\lambda}, \tag{3.8c}$$

$$\sigma_1 = -\lambda. \tag{3.8d}$$

This is an exact solution to the gap equation (3.7) when the system attains its maximum Casimir energy. In this simple case we can obtain formulas for ρ^2 and V as functions of the dimensionless mass parameter μ. From (2.13), (3.8a) and (3.8c) we see that

$$\lambda = 1 - \frac{\mu^2 s_0^2}{\rho^2}. \tag{3.9}$$

Comparing this with (2.15) we find

$$\rho = \mu s_0\sqrt{1 - \frac{(D-2)}{2\pi}\frac{\eta(s_0)}{\mu^2 s_0^2}}. \tag{3.10}$$

The potential can then be written as a function of μ as follows:

$$\overline{V}(\mu) = \mu s_0 \sqrt{1 + \frac{(D-2)}{2\pi} \frac{\eta(s_0)}{\mu^2 s_0^2}}. \qquad (3.11)$$

Since $\eta(s_0) < 0$, (3.11) implies that there is a minimum value of μ for which the potential exists at $s = s_0$, denoting this value by μ_{min} we see that it is given by

$$\mu_{min} = \sqrt{\frac{(D-2)}{2\pi} \frac{|\eta(s_0)|}{s_0^2}}. \qquad (3.12)$$

The general solution of the problem is given by (3.5) and (3.7). Figure 4 shows the behaviour of α_0, α_1, σ_0 and σ_1 as functions of ρ for various values of the mass parameter μ. We see that for large ρ, α_0 and α_1 tend to unity, whereas σ_0 and σ_1 approach zero. Thus from (2.13) we see that for finite μ a large ρ is equivalent to a large s; this will be of interest when discussing the Lüscher term in the following section. In Fig. 5 we show the behaviour of the potential $\overline{V}(\rho)$ for several values of the mass μ. We see that for big and small values of μ the curves come close together, in agreement with (2.12), approaching the Nambu–Goto result for $\mu = 0, \infty$. We also see that the small bump in Fig. 2 of [17] for $\mu \approx 0.3$ is not present. This is probably a numerical artifact. We next show in Fig. 6 the so-called deconfinement radius ρ_{dec} as a function of μ. This is the value of ρ for which the potential vanishes and probably signals the presence of the tachyon in string models. Comparing this with Fig. 3 of [17] we see that the behaviour is very similar, avoiding, however, the numerical trick of [17] at $\mu \approx 0.1$. Thus we have performed an accurate numerical investigation of the exact gap equations for the Nambu–Goto string model with point masses attached to its ends. There is a substantial departure from the usual Nambu–Goto model with fixed ends (i.e., infinitely heavy quarks masses). In particular the deconfinement radius can take a whole range of values, allowing for a better phenomenological description of mesonic systems. On the other hand the typical linear behaviour for large ρ is mantained here.

All these results can be seen in Fig. 5, where the interquark static potential for various μ mass values is compared with the usual Nambu–Goto model.

4 Discussion and conclusions

We have obtained exact results to the problem of quark mass corrections to the string potential for the Nambu–Goto model in the case where the masses attached to the ends of the string are equal. These results are similar to those presented by Lambiase and Nesterenko [17] obtained under some symplifying assumptions. There is, however, a subtle point concerning the Lüscher term, which we would like to discuss. For a string with fixed ends, the Lüscher term has a contribution to the potential of the form

$$\overline{V}_{\mathrm{L}}(\rho) = -\frac{(D-2)\pi}{24\rho}. \qquad (4.1)$$

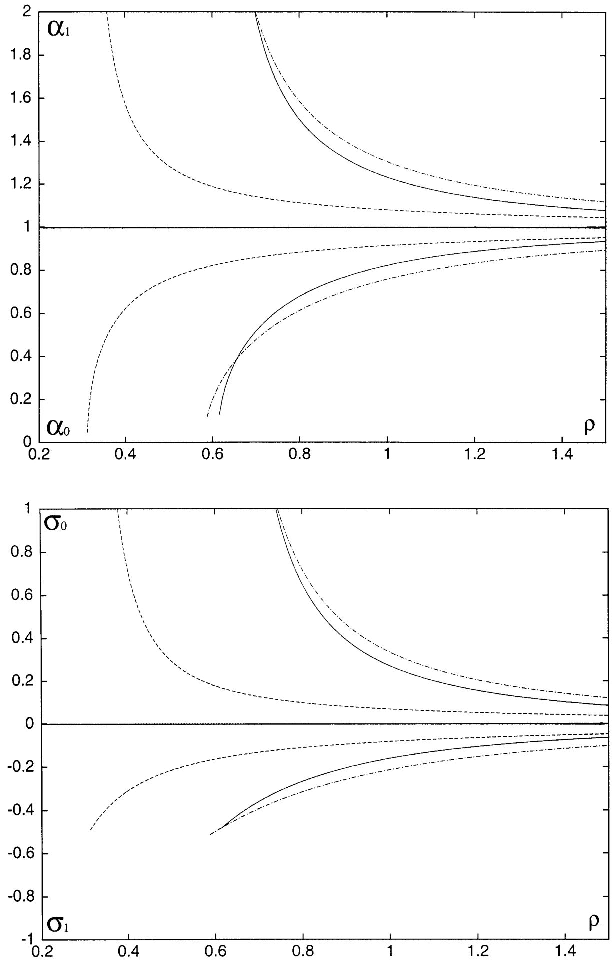

Fig. 4. The solutions to the gap equations (3.7) for the Lagrange multipliers α_0, α_1 and metric components σ_0, σ_1 are shown as functions of $\rho = M_0 R$ for $\mu = 100, 1$, and 0.1 (solid, dashed and dash-dotted lines, respectively). In **a** the curves for α_0 are the lower ones and for α_1 the upper ones, while those for σ_0 (above) and σ_1 (below) appear in **a**. The minimum value α_0 can reach is zero as follows from (2.21)

The importance of this term is that it is universal, i.e., independent of the details of a whole class of models, in particular, independent of the parameters of the model under consideration. In the one-loop approximation to the problem discussed above the potential becomes

$$\overline{V}(\rho) = \rho + (D-2)E_{\mathrm{C}}, \qquad (4.2)$$

where E_{C} is given by

$$E_{\mathrm{C}} = \frac{\eta(s)}{2\pi\rho}, \qquad (4.3)$$

and E_{C} depends on the mass μ through s (see (2.13)) thus apparently giving the Lüscher term a mass dependence. It is important to note, however, that this Coulomb-like term arises as a long-distance (large-ρ) effect. Thus strictly speaking corrections to the Lüscher term, if any, should be obtained after expanding (4.2) for large ρ. From our numerical results we can see that a large ρ is equivalent

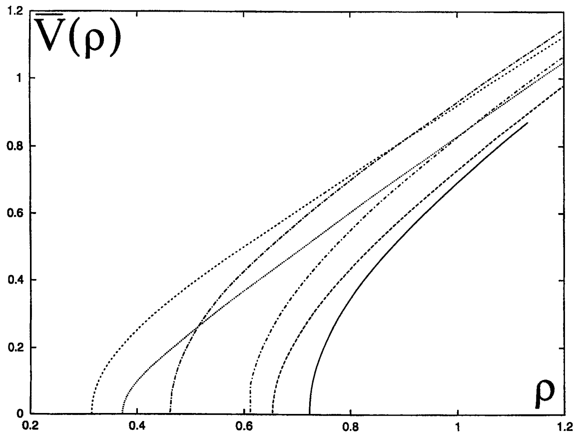

Fig. 5. The dimensionless interquark potential $\overline{V}(\rho)$, (2.20), is shown as a function of $\rho = M_0 R$ for $\mu = 1.15$, 0.3, 10, 100, 0.05, 0 or ∞ (from left to right) at the interception with the x axis (short-dashed, dotted, dash-dotted, short-dash-dotted, dashed and solid lines, respectively). When $\mu = 0, \infty$ (dash-dotted line) corresponding to free and fixed ends strings respectively the potential becomes the well known Nambu–Goto potential. As the mass μ varies between zero and infinity $\overline{V}(\rho)$ essentially keeps its shape but reaches a vanishing value at different deconfinement radii ρ_{dec} (see Fig. 6). In all the cases the potential becomes linear for large values of ρ. The lines second and third from left differ notably from the results of [17], although in [18] the curve with $\mu = 10$ (third) has been corrected and looks the same as the one presented above. For small and big values of μ the approximation of [17] seems to be good

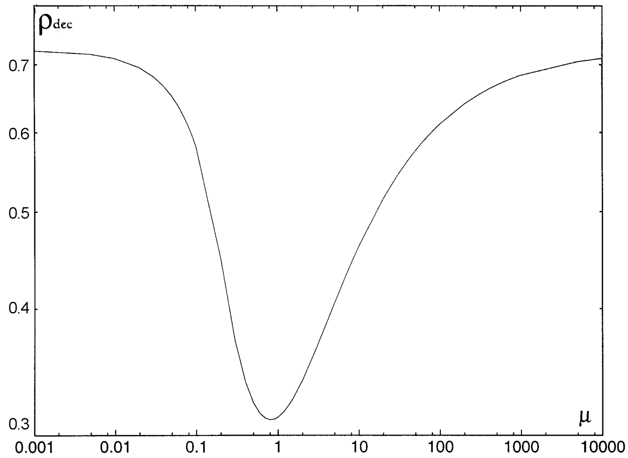

Fig. 6. The so-called deconfinement radious $\rho_{\rm dec}$ which is defined as the value of ρ for which $\overline{V}(\rho = \rho_{\rm dec}) = 0$ is here shown as a function of the mass parameter $\mu = m/M_0$. For $\mu = 0, \infty$ $\rho_{\rm dec}(\mu) = \sqrt{(D-2)\pi/12}|_{D=4} \approx 0.72$ as in the Nambu–Goto case. For finite μ values $\rho_{\rm dec}$ lies in the interval $0.31 \le \rho_{\rm dec} \le \sqrt{(D-2)\pi/12}|_{D=4} \approx 0.72$

to a large s for a given finite value of μ. Thus for large ρ, α_0 and α_1 are essentially one and from (2.23) and (2.24) the potential becomes

$$\overline{V}(\rho) \approx \rho\sqrt{1-2\lambda} \approx \rho(1-\lambda+...). \qquad (4.4)$$

For large s we can approximate the integral involved in the definition of λ, (2.20), with the result

$$\lambda \approx \frac{(D-2)\pi}{24\rho^2} - \frac{(D-2)\pi}{12\rho^3}\mu, \qquad s \to \infty. \qquad (4.5)$$

Thus the potential becomes

$$\overline{V}(\rho) = \rho - \frac{(D-2)\pi}{24\rho} + \frac{(D-2)\pi}{12\rho^2}\mu + ..., \qquad (4.6)$$

leaving the Lüscher term universal.

So the study of the interquark potential for string models with masses attached to its ends is of undoubted interest by itself as a mathematical problem and certainly for the possible physical applications to the low-energy regime of QCD. Here we have presented exact solutions to the gap equations and the interquark potential has been obtained for several values of μ. We see that having finite point masses at the ends of the string has considerable effects on the potential. Also the deconfinement radius become a function of μ and its value could be fixed phenomenologically. We have also discussed the universality of the Lüscher term and argue that it remains universal if we understand it strictly as a long-distance effect with mass corrections coming up at higher orders in ρ^{-1}. Finally, the tachyon problem of string theories remains unresolved although recently [19] there has been some discussion on how one can possibly avoid it. The tachyon problem, however, would affect the short-distance behaviour, where the string picture breaks down in any case. For large distances and for any number of dimensions the string anomaly disappears asymptotically since the anomalous term is accompanied by a ρ^{-2} factor [20], leaving a consistent string model in that regime. For a recent discussion of this point see [21].

Acknowledgements. We are grateful to Dr. R. Stuart for useful discussions.

References

1. Y. Nambu, in *Symmetries and Quark Models*, edited by R. Chand (Gordon and Breach, New York, 1970); T. Goto, Prog. Theor. Phys. **46**, 1560 (1971)
2. T. Eguchi, Phys. Rev. Lett. **44**, 126 (1980)
3. A. Schild, Phys. Rev. D **16**, 1722 (1977)
4. M. Lüscher, K. Symanzik, P. Weisz, Nucl. Phys. **B173**, 365 (1980)
5. O. Alvarez, Phys. Rev. D **24**, 440 (1981)
6. J.F. Arvis, Phys. Lett. B **127**, 106 (1983)
7. R.D. Pisarski, O. Alvarez, Phys. Rev. D **26**, 3735 (1982); A. Antillón, G. Germán, Phys. Rev. D **47**, 4567 (1993)
8. M. Flensburg, C. Peterson, Nucl. Phys. B **283**, 141 (1987); F. Karsch, E. Laermann, Rep. Prog. Phys. **56**, 1347 (1993)
9. J. Polchinski, Phys. Rev. Lett. **68**, 1267 (1992); Phys. Rev. D **46**, 3667 (1992)
10. G. Germán, H. Kleinert, M. Lynker, Phys. Rev. D **46**, 1699 (1992); G. Germán, M. Lynker, A. Macías, Phys. Rev. D **46**, 3640 (1992); G. Germán, M. Lynker, Phys.

Rev. D **46**, 5678 (1992); A. Antillón, G. Germán, Phys. Rev. D **49**, 1966 (1994)
11. M. Natsuume, Phys. Rev. D **48**, 835 (1993)
12. A.M. Polyakov, Nucl. Phys. **B268**, 406 (1986); H. Kleinert, Phys. Lett. **174B** 335 (1986); for a review see A. M. Polyakov, *Gauge Fields and Strings* (Harwood, Chur, 1987)
13. G. Germán, Mod. Phys. Lett. A **6**, 1815 (1991)
14. M. Awada, Phys. Lett. B **351**, 468 (1995)
15. A.M. Polyakov, PUPT-1632, hep-th/9607049; M.C. Diamantini, F. Quevedo, C.A. Trugenberger, Phys. Lett. B **396**, 115 (1997)
16. V.V. Nesterenko, Z. Phys. C **51**, 643 (1991); V.V. Nesterenko, N.R. Shvetz, Z. Phys. C **55**, 265 (1992); B.M. Barbashov, V.V. Nesterenko, *Introduction to the Relativistic String Theory* (Worl Scientific, Singapore, 1990)
17. G. Lambiase, V.V. Nesterenko, Phys. Rev. D **54**, 6387 (1996)
18. G. Lambiase, V.V. Nesterenko, Phys. Rev. D **55**, 6603 (1997)
19. H. Kleinert, G. Lambiase, V.V. Nesterenko, Phys. Lett. B **384**, 213 (1996)
20. P. Olesen, Phys. Lett. B **160**, 144 (1985)
21. T.J. Allen, M.G. Olsson, S. Veseli, Phys. Lett. B **434**, 110 (1998)